

The mass–temperature relation for clusters of galaxies

Jens Hjorth,^{1,2★} Jamila Oukbir^{3★} and Eelco van Kampen^{4★}

¹*NORDITA, Blegdamsvej 17, DK–2100 Copenhagen Ø, Denmark*

²*Centre for Advanced Study, Drammensveien 78, N–0271 Oslo, Norway*

³*Danish Space Research Institute, Juliane Maries Vej 30, DK–2100 Copenhagen Ø, Denmark*

⁴*Theoretical Astrophysics Center, Juliane Maries Vej 30, DK–2100 Copenhagen Ø, Denmark*

Accepted 1998 February 25. Received 1998 February 18; in original form 1997 October 10

ABSTRACT

A tight mass–temperature relation, $M(r)/r \propto T_X$, is expected in most cosmological models if clusters of galaxies are homologous and the intracluster gas is in global equilibrium with the dark matter. We here calibrate this relation using eight clusters with well-defined global temperatures measured with *ASCA* and masses inferred from weak and strong gravitational lensing. The surface lensing masses are deprojected in accordance with *N*-body simulations and analytic results. The data are well-fitted by the mass–temperature relation and are consistent with the empirical normalization found by Evrard et al. (1996) using gas-dynamic simulations. Thus, there is no discrepancy between lensing and X-ray-derived masses using this approach. The dispersion around the relation is 27 per cent, entirely dominated by observational errors. The next generation of X-ray telescopes combined with wide-field *HST* imaging could provide a sensitive test of the normalization and intrinsic scatter of the relation, resulting in a powerful and expedient way of measuring masses of clusters of galaxies. In addition, as $M(r)/r$ (as derived from lensing) is dependent on the cosmological model at high redshift, the relation represents a new tool for determination of cosmological parameters, notably the cosmological constant Λ .

Key words: celestial mechanics, stellar dynamics – galaxies: clusters: general – cosmology: observations – cosmology: theory – dark matter – gravitational lensing.

1 INTRODUCTION

Clusters of galaxies are the largest gravitationally bound structures in the Universe and are as such excellent probes of cosmic structure formation and evolution. The ensemble properties of clusters expected in various cosmological scenarios can be used to derive constraints on the power spectrum of the initial density perturbations and on cosmological parameters such as Ω_0 and Λ (e.g. Eke, Cole & Frenk 1996; Bahcall, Fan & Cen 1997; Oukbir & Blanchard 1997; Bartelmann et al. 1998; de Theije, van Kampen & Slijkhuis 1998). On the scales of individual clusters the inferred baryon mass fraction can be used to constrain Ω_0 (White et al. 1993; Evrard 1997). In such studies, an important quantity is the total cluster mass or any observed quantity which is tightly related to the mass.

A promising mass estimator is the mean emission-weighted temperature, T_X , of the hot intracluster medium (ICM) in clusters of galaxies. Based on numerical simulations, it has been shown that T_X is a better indicator of the total mass of a cluster than any other optical or X-ray property (Evrard 1990). Recently, Evrard, Metzler & Navarro (1996, hereafter EMN) and Eke, Navarro & Frenk

(1997) showed that there is a tight relation between the mass of a cluster and its global X-ray temperature in cosmological gas-dynamic simulations, irrespective of the state of the cluster (e.g. not restricted to clusters with a ‘regular’ appearance or ‘isothermal’ clusters) and the assumed cosmological model. In the simulations, it was found that mass predictions using this method (which only involve temperatures) are twice as precise as those derived using the β model (which require the surface brightness distribution in addition, i.e. more photons and higher spatial resolution). However, the normalization of the relation hinges on numerical simulations which may not comprise sufficient detail (EMN; Anninos & Norman 1996). Therefore, it is essential to calibrate this relation from an observational point of view, by using independent mass estimators.

The purpose of this Letter is to provide a first observational calibration of the M – T_X relation using a relatively ‘clean’ way of determining independent cluster masses by gravitational lensing. This technique essentially probes the projected mass along the line of sight. It is also pointed out that the relation holds the promise of providing a test of the geometry of the Universe, which is particularly sensitive to Λ . Throughout this Letter, however, we assume a standard homogeneous Einstein–de Sitter Universe with $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$, $h = 0.5$, $\Omega_0 = 1$ and $\Lambda = 0$.

★E-mail: jens@nordita.dk (JH); jamila@dsri.dk (JO); eelco@tac.dk (EvK)

2 THE MASS–TEMPERATURE RELATION

Navarro, Frenk & White (1997, hereafter NFW) found in their numerical simulations that the dark matter distribution in present-day clusters has self-similar density profiles when the radial coordinate is scaled to the radius containing an overdensity of $\delta = 200$ relative to the critical density. More precisely, defining the overdensity as

$$\delta(r_\delta, z) \equiv \frac{3M_\delta(r_\delta)}{4\pi\rho_c(z)r_\delta^3}, \quad (1)$$

where $\rho_c(z) = \rho_{c0}(1+z)^3$ and $\rho_{c0} = 3H_0^2/(8\pi G)$, NFW found that clusters are well-described by the density profile $\rho(x) \propto x^{-1}(1+cx)^{-2}$, where $x = r/r_{200}$, in any cosmology. The variation in c ($\approx 5-10$) with mass, cosmological parameters, and redshift is small (Cole & Lacey 1996; NFW; Bartelmann et al. 1998; Eke, Navarro & Frenk 1997) and so clusters form a homologous family to a good approximation when scaled to a given overdensity. Optical (Carlberg et al. 1997) and lensing observations (Fischer & Tyson 1997) seem to support this conclusion.

For a cluster in quasi-equilibrium (Natarajan, Hjorth & van Kampen 1997) the virial theorem for the dark matter states that $M(r) \propto r\langle v^2 \rangle_r$. Self-similarity implies that the constant of proportionality depends on the adopted overdensity only. Finally, the global X-ray temperature is assumed to be proportional to the global mean velocity dispersion of the dark matter (at any time), i.e. $T_X \propto \langle v^2 \rangle_r$. For example, this would be the case in the absence of transient effects and non-gravitational heating or cooling effects. In the case of equipartition, one would have a universal $\beta \equiv \mu m_p \langle v^2 \rangle_r / (kT) = 1$. Combining these assumptions (quasi-equilibrium, self-similarity, proportionality between dark-matter velocity dispersion and X-ray temperature) leads to a simple scaling relation between the characteristic mass and radius at a given overdensity, δ , and the global emission-weighted temperature of the hot X-ray gas,

$$M_\delta(r_\delta) = k_\delta r_\delta T_X, \quad (2)$$

where k_δ is a constant depending on δ . This equation expresses the structural invariance of clusters under mass and redshift transformations and *does not* rely on any particular dark-matter density profile or the assumption of hydrostatic equilibrium.

Combined with the definition of the overdensity (equation 1), this expression leads to the mass–temperature relation

$$M_\delta = k_\delta^{3/2} \left(\frac{3}{4\pi\delta\rho_{c0}} \right)^{1/2} \left(\frac{T_X}{1+z} \right)^{3/2}, \quad (3)$$

or, equivalently, the size–temperature relation

$$r_\delta(1+z) = k_\delta^{1/2} \left(\frac{3}{4\pi\delta\rho_{c0}} \right)^{1/2} \left(\frac{T_X}{1+z} \right)^{1/2}, \quad (4)$$

where $r_\delta(1+z)$ is the comoving angular radius of the cluster.

Rather than trying to compute the prefactor k_δ from first principles, EMN used numerical simulations to calibrate these relations. They found the radius r_{500} to be a conservative estimate of the boundary between the virialized region of the clusters and their outer envelopes. At $z = 0.04$ using $\delta = 500$ they found a universal prefactor independent of Ω_0 ,

$$M_{500} = 2.22 \times 10^{15} \left(\frac{T_X}{10 \text{ keV}} \right)^{3/2} M_\odot. \quad (5)$$

In the simulations the scatter around this relation was found to be only 15 per cent compared with 30 per cent when using the β model to estimate the mass.

Mohr & Evrard (1997) have recently shown that observations of nearby clusters lead to an intrinsic scatter of 10–15 per cent in the relation between cluster isophotal size and mean emission-weighted temperature T_X (similarly to equation 4) regardless of the state of the cluster (merging, cooling flow) thus giving added support to the existence of a tight mass–temperature relation.

3 LENSING MASSES

In order to test and calibrate the mass–temperature relation observationally we shall use independent masses determined from gravitational lensing. As lensing masses are given in the literature as a function of physical radius rather than overdensity, we shall use equation (2) to express the temperature as a function of $M(r)/r$ instead of M_δ . Thus the relation we shall test observationally is

$$\left(\frac{M(r)}{10^{15} M_\odot} \right) \left(\frac{1 \text{ Mpc}}{r} \right) = k_\delta \left(\frac{T_X}{10 \text{ keV}} \right). \quad (6)$$

For an isothermal sphere, $M(r) \propto r$, k_δ would be a constant independent of radius or overdensity. However, given the fact that clusters are described by more complicated density and temperature profiles (NFW; EMN), k_δ varies slightly with δ in the range considered. While an overdensity of ~ 500 was recommended ($r_{500} \sim 1-2$ Mpc), EMN provided normalizations for $\delta = 100, 250, 500, 1000$ and 2500 . Converting these into equivalent values for k_δ we find 0.76, 0.91, 1.01, 1.09 and 1.14. These slowly varying numbers are used to compute k_δ as a function of δ by spline interpolation.

3.1 Deprojection

Lensing provides the 2D projected (surface) mass, $M_{2D}(R)$ (where R indicates a projected radius), of the cluster. In general the 2D mass at a given radius is larger than the 3D mass, $M_{3D}(r)$, evaluated at the same radius ($r = R$). The best way of obtaining a deprojection relation for M_{3D} (which is the quantity entering equation 6) is to study numerical simulations of galaxy clusters, preferably a fair sample of these. We have used the catalogue of simulated standard CDM clusters ($\Omega = 1$) of van Kampen & Katgert (1997) to find such a relation (van Kampen, in preparation). As the clusters we shall study in this Letter are biased towards massive clusters, we selected only clusters with a total mass within the Abell radius (3 Mpc) of at least $10^{15} M_\odot$. The deprojection relation $M_{3D}(x)/M_{2D}(x)$ for these 41 clusters is plotted as a function of dimensionless radius $x = r/r_{200}$ in Fig. 1. The scatter in the relation is fairly substantial at small radii as substructure along the line of sight becomes important for the projected mass. In an open $\Omega_0 = 0.2$ CDM Universe the corresponding curve (not plotted) is 10 per cent higher because of the smaller influence of substructure.

In Fig. 1 we also plot the deprojection relation of the Hernquist (1990) and the Brainerd, Blandford & Smail (1996, hereafter BBS) models, which both have analytic deprojection properties [the BBS model is the limit of $\eta \rightarrow \infty$ of the Hjorth & Kneib (1998) model]. As the total mass of the NFW model is infinite it is not useful for this purpose. However, we have made use of the fact that the half-mass radius in the Hernquist model is roughly equivalent to r_{200} in the NFW model for $c \approx 5$ (Cole & Lacey 1996). In the outer parts the Hernquist and BBS models coincide, but in the centre there is a marked difference between the two curves because of their differing divergence properties. The Hernquist model, which diverges as $\rho \sim r^{-1}$ in the centre, is similar to the NFW profile and $M_{3D}(x)/M_{2D}(x) \rightarrow 0$ for $x \rightarrow 0$, while the BBS model, which has a

stronger central cusp $\rho \sim r^{-2}$, tends to the value for the singular isothermal sphere $2/\pi \approx 0.64$. This shows that deprojection of 2D masses at small radii depends sensitively on the exact slope of the inner cusp of dark matter density profiles (Fukushige & Makino 1997; Moore et al. 1998; Kravtsov et al. 1998). We finally note that the Hernquist model is in excellent agreement with the numerical results of the open model.

In this Letter we shall use the relation as a function of proper radius R to deproject the lensing masses. For this purpose we introduce a convenient fitting function,

$$\frac{M_{3D}}{M_{2D}}(R) = 0.56 \tan^{-1} \left(\frac{R}{0.28 \text{ Mpc}} \right), \quad (7)$$

where the coefficients have been determined from a non-linear least-squares fit up to $R = 2$ Mpc. We note that if such a deprojection correction is not applied, lensing (2D) masses will be higher than X-ray (3D) masses by a factor of 1.5 on average.

4 DATA

We have compiled a list of clusters with well-determined X-ray temperatures and masses determined independently using gravitational lensing. The data are shown in Table 1.

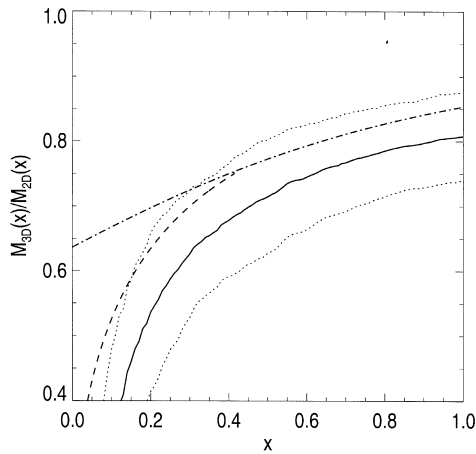


Figure 1. This figure shows $M_{3D}(x)/M_{2D}(x)$ as a function of $x = r/r_{200}$ for the Hernquist (1990) profile (dashed curve) and the model of BBS (dash–dotted curve). The solid curve is the corresponding mean deprojection factor for simulated massive clusters in a CDM $\Omega = 1$ Universe (see text for details) and the dotted curves indicate the 1σ confidence interval. The corresponding curve for an open CDM $\Omega_0 = 0.2$ Universe (not plotted here) lies 10 per cent higher, in good agreement with the Hernquist model.

Table 1. Observational data on clusters with ASCA temperatures and lensing masses. The temperatures are from Mushotzky & Scharf (1997). The lensing masses are generally taken from the most recent publication of a given cluster. For the data of Smail et al. (1995) we have assumed an uncertainty of 40 per cent in the masses and adopted the no evolution model for the redshift distribution of faint background galaxies. For the data of Squires et al. (1996a,b, 1997) we have estimated the masses inside 210 arcsec. All masses and radii are computed assuming $h = 0.5$, $\Omega_0 = 1$ and $\Lambda = 0$.

Cluster	z	T_X (keV)	R (Mpc)	$M_{2D}(R)$ ($10^{14} M_\odot$)	$M_{3D}(R)$ ($10^{14} M_\odot$)	δ	k_δ	Reference
Abell 2218	0.17	$7.48^{+0.53}_{-0.41}$	0.80	9.4 ± 1.7	6.5 ± 1.2	2714	1.14	Squires et al. (1996a)
Abell 1689	0.18	$9.02^{+0.40}_{-0.30}$	0.48	10.0 ± 1.8	5.8 ± 1.1	11057	1.15	Taylor et al. (1998)
Abell 2163	0.20	$12.7^{+2.0}_{-2.0}$	0.90	13.0 ± 10	9.2 ± 7	2524	1.14	Squires et al. (1997)
Abell 2390	0.23	$8.90^{+0.97}_{-0.77}$	0.95	10 ± 4	7.2 ± 2.9	1551	1.14	Squires et al. (1996b)
MS 1455.0+2232	0.26	$5.45^{+0.29}_{-0.28}$	0.45	3.6 ± 1.4	2.0 ± 0.8	3859	1.14	Smail et al. (1995)
MS 1358.4+6245	0.33	$6.50^{+0.68}_{-0.64}$	1.00	4.4 ± 0.6	3.2 ± 0.4	468	1.00	Hoekstra et al. (1998)
RX J1347–1145	0.45	$11.37^{+1.10}_{-0.92}$	2.00	34 ± 8	27 ± 6	385	0.98	Fischer & Tyson (1997)
MS 0015.9+1609	0.54	$8.0^{+1.0}_{-1.0}$	0.60	8.5 ± 3.4	5.4 ± 2.2	2355	1.14	Smail et al. (1995)

The X-ray data used here are from a recent compilation of temperatures of intermediate- and high-redshift clusters observed by the *Advanced Satellite for Cosmology and Astrophysics* (ASCA) (Mushotzky & Scharf 1997). The temperatures were measured in a uniform way out to a radius of 3–6 arcmin depending on the redshift of the cluster.

The lensing masses are from various studies of individual clusters, mostly using the ‘weak lensing’ method pioneered by Kaiser & Squires (1993), but also from variations in number counts of background galaxies (Broadhurst, Taylor & Peacock 1995; van Kampen 1998). We included only clusters with masses determined out to radii larger than 400 kpc to minimize deprojection and substructure effects from the central regions of the clusters. One cluster mass (MS 1358+62) was determined from wide-field *HST* imaging. The normalization of the mass of A2163 was adjusted in comparison with X-ray masses (derived from the β model), i.e. this mass is not completely independent of the temperature (Squires et al. 1997).

We show the results for the eight clusters in Fig. 2, in which we plot $M_{3D}(R)/R$ derived from lensing studies as a function of $k_\delta T_X$. The relation predicted by equation (6) gives an excellent fit to the data. The best-fitting line has a normalization which is 88 per cent of that predicted by EMN and the dispersion (rms) about the relation is 27 per cent in mass, somewhat smaller than that expected from the quoted observational errors alone.

5 DISCUSSION

The observed scatter around the mass–temperature relation (equation 6) is dominated by observational errors and is consistent with having no intrinsic scatter. The data thus support the existence of an $M-T_X$ relation as a fairly accurate independent estimator of cluster masses. It is, however, important to point out that the data set presented here may be affected by systematic errors in both the masses and the temperatures. Adopting different data sets could lead to significant changes in e.g. the normalization of the mass–temperature relation.

X-ray temperatures of hot clusters are usually uncertain because of the few photons detected above 8 keV with ASCA, and quoted errors normally do not incorporate possible systematic errors. Thus, many compilations (e.g. Sadat, Blanchard & Oukbir 1998) may be affected by the fact that temperatures often differ from author to author (Mushotzky & Scharf 1997; Allen 1998; Yamashita 1997) because of differences in data analysis and use of data from other satellites (*ROSAT*, *Ginga*). For example, Allen (1998) has shown that cooling flows may bias global temperatures as derived by

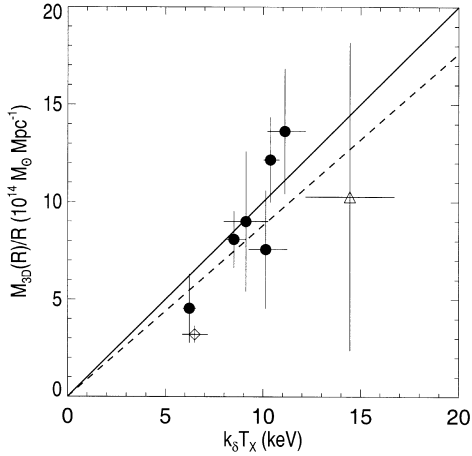


Figure 2. This figure shows the $M_{3D}(R)/R-T_X$ relation for lensing clusters of galaxies. Filled circles are ground-based data, the open diamond is the *HST* data point of MS 1358+62 and the open triangle is A2163. The error bars do not include deprojection errors. The solid line is the relation predicted by equation (6) as normalized by EMN and has not been fitted to the data. The dashed line minimizes the mean relative residual and has a normalization 12 per cent lower.

Mushotzky & Scharf (1997) downward by about 30 per cent on average. In this Letter we have used the Mushotzky & Scharf (1997) data because of the uniformity and simplicity of the analysis and the possibility of extending it to fainter and higher redshift clusters.

Ground-based lensing data are affected by fairly large seeing corrections. *HST* results are therefore preferable but usually only available inside a small radius. The normalization of weak-lensing masses (owing to the mass-sheet degeneracy) can be carried in various ways, using e.g. multiple arcs, magnification bias, fits of analytic models or comparison with X-ray profiles. Sadat et al. (1998) found that a normalization of k_{δ} about 64 per cent of the EMN value (no deprojection was applied) was consistent with the temperatures and masses of five *HST* clusters studied out to 400kpc by Smail et al. (1997). Such a small normalization would imply that the data presented here have systematically overestimated masses or underestimated temperatures. The high-quality *HST* data point for MS 1358+62 may indicate that ground-based masses are indeed overestimated. However, the mass of MS 1358+62 as derived from weak lensing could also be underestimated e.g. due to deprojection errors arising because of the high ellipticity of the cluster. Such an underestimate of the 3D mass is supported by the disagreement between the velocity dispersion derived from weak lensing ($780 \pm 50 \text{ km s}^{-1}$) and that found from direct spectroscopic measurements, as well as from strong lensing ($\sim 1000 \text{ km s}^{-1}$) (Hoekstra et al. 1998). *HST* clusters may also have underestimated masses, because of the fact that a fit of an isothermal sphere to the mean tangential shear inside a small radius (Smail et al. 1997) in general biases masses low (van Kampen & Hjorth, in preparation).

It is a long-standing discussion whether masses determined from lensing agree with or exceed X-ray masses determined using the β model (see e.g. Smail et al. 1997; Allen 1998). If we take the results presented in Fig. 2 at face value, the good agreement between the EMN normalization and the observational calibration indicates that there is no such discrepancy when using the mass–temperature relation. If anything the X-ray masses computed using the EMN normalization are slightly higher (by ~ 10 –20 per cent; cf. Fig. 2 and MS 1358+62) than lensing masses. Such an effect would be

consistent with the predictions of simulations incorporating the effects of galactic winds (Metzler & Evrard 1998) which contribute to heating the ICM.

Besides its use as a straightforward mass estimator for any cluster with a well-determined global temperature, the mass–temperature relation holds the promise of becoming an important cosmological tool, bearing a resemblance to the Fundamental Plane or Tully–Fisher scaling relation for elliptical or spiral galaxies, respectively, in that it relies on simple scaling relations with 10–20 per cent scatter, but presumably involves much smaller evolutionary corrections. A direct cosmological application of the $M-T_X$ relation would be to examine the inferred deviations from it as a function of redshift. A possible trend with redshift could be indicative of (i) evolutionary effects, (ii) the assumed redshift distribution of the faint background galaxies, $N(z)$, or (iii) the parameters entering the assumed cosmological model.

Typical evolutionary effects could be non-gravitational heating or cooling of the ICM, such as effects of feedback mechanisms like galactic winds which introduce systematic structural changes of the ICM (Metzler & Evrard 1998) or cooling flows (Allen 1998). Possible ‘outliers’ from the relation could be due to e.g. merging clusters with a very unsettled temperature distribution (Schindler 1996) or highly elongated clusters which give large deprojection uncertainties depending on the viewing angle.

While the inferred lensing masses of low- and intermediate-redshift clusters are fairly insensitive to the assumed median redshift of the background galaxies, high-redshift clusters are very sensitive to the assumed median redshift (Smail et al. 1995; Luppino & Kaiser 1997) and so deviations from the expected relation at high redshift could be used to constrain $N(z)$.

Finally, the world model enters through the derived masses and sizes via the expression for the angular diameter distance. In the simplest form, $T_X \propto D_S/D_{LS}$, where T_X is a directly measurable intrinsic quantity and D_S/D_{LS} is the ratio between the source and lens–source angular diameter distances. Thus, the method can be used as a test for the geometry of the Universe, which is less sensitive to inhomogeneities along the line of sight than small standard rods/candles (e.g. SN Ia) (Hadrović & Binney 1997). Individual massive high-redshift clusters could therefore be fairly unbiased discriminators between different cosmological models, particularly sensitive to the cosmological constant Λ . At $z = 1$ the difference between a $(\Omega_0, \Omega_{\Lambda}) = (1, 0)$ and a $(\Omega_0, \Omega_{\Lambda}) = (0.2, 0.8)$ Universe is 25 per cent in D_S/D_{LS} . Moreover, if the measurement of the global X-ray temperature is supplemented with spatially resolved X-ray imagery additional constraints on Ω_0 can be derived (Sasaki 1996; Pen 1997).

6 CONCLUSION

Based on numerical simulations (EMN; Eke et al. 1997) and observations of nearby clusters (Mohr & Evrard 1997) the existence of a tight mass–temperature relation has been suggested. The results presented here provide support for this assertion and indicate that the mass–temperature relation (equation 6) can be used to determine cluster masses with a precision of 27 per cent (Fig. 2). There seems to be no significant discrepancy between deprojected lensing masses and masses derived from X-ray temperatures, using the normalization found in numerical simulations (EMN).

The origin of this tight relation is believed to be the fairly simple physics entering the relation (cf. Section 2), namely virialization of gravitationally bound structures with self-similar dark matter density distributions that are in global quasi-equilibrium with the hot

ICM, independent of the chosen world model, power spectrum or exact formation redshift of the cluster.

We have cautioned that the observational data discussed in this Letter are quite uncertain and possibly affected by systematic errors. The results should therefore only be taken as an indication of a tight mass–temperature relation. However, the future observational situation is promising. A sample of clusters with very precise lensing masses (e.g. from wide-field *HST* imaging with the ACS) to about 10 per cent or better (e.g. Natarajan et al. 1998; Hoekstra et al. 1998) and equally accurate temperatures (e.g. with *AXAF*, *Spectrum-XG* or *XMM*) would allow us to study the intrinsic scatter of the relation and determine a precise normalization. This could provide a direct and reliable mass estimator for distant clusters with important cosmological implications.

ACKNOWLEDGMENTS

We thank Monique Arnaud, Henk Hoekstra, Jean-Paul Kneib and Ian Smail (the referee) for useful comments and discussions. JH acknowledges the hospitality of DSRF where part of this work was carried out. This work was supported in part by Danmarks Grundforskningsfond through its funding of the Theoretical Astrophysics Center.

REFERENCES

- Allen S. W., 1998, *MNRAS*, 296, 392
 Anninos P., Norman M. L., 1996, *ApJ*, 459, 12
 Bahcall N. A., Fan X., Cen R., 1997, *ApJ*, 485, L53
 Bartelmann M., Huss A., Colberg J. M., Jenkins A., Pearce F. R., 1998, *A&A*, 330, 1
 Brainerd T. G., Blandford R. D., Smail I., 1996, *ApJ*, 466, 623 (BBS)
 Broadhurst T. J., Taylor A. N., Peacock J. A., 1995, *ApJ*, 438, 49
 Carlberg R. G. et al., 1997, *ApJ*, 485, L13
 Cole S., Lacey C., 1996, *MNRAS*, 281, 716
 de Theije P., van Kampen E., Slijkhuis R., 1998, *MNRAS*, submitted
 Eke V. R., Cole S., Frenk C. S., 1996, *MNRAS*, 282, 263
 Eke V. R., Navarro J. F., Frenk C. S., 1997, *ApJ*, in press (astro-ph/9708070)
 Evrard A. E., 1990, *ApJ*, 363, 349
 Evrard A. E., 1997, *MNRAS*, 292, 289
 Evrard A. E., Metzler C. A., Navarro J. F., 1996, *ApJ*, 469, 494 (EMN)
 Fischer P., Tyson J. A., 1997, *AJ*, 114, 14
 Fukushige T., Makino J., 1997, *ApJ*, 477, L9
 Hadrović F., Binney J., 1997, *MNRAS*, submitted (astro-ph/9708110)
 Hernquist L., 1990, *ApJ*, 356, 359
 Hjorth J., Kneib J.-P., 1998, *ApJ*, submitted
 Hoekstra H., Franx M., Kuijken K., Squires G., 1998, *ApJ*, in press (astro-ph/9711096)
 Kaiser N., Squires G., 1993, *ApJ*, 404, 441
 Kravtsov A. V., Klypin A. A., Bullock J. S., Primack J. R., 1998, *ApJ*, in press (astro-ph/9708176)
 Luppino G. A., Kaiser N., 1997, *ApJ*, 475, 20
 Metzler C. A., Evrard A. E., 1998, *ApJ*, submitted (astro-ph/9710324)
 Mohr J. J., Evrard A. E., 1997, *ApJ*, 491, 38
 Moore B., Governato F., Quinn T., Stadel J., Lake G., 1998, *ApJ*, 499, L5
 Mushotzky R. F., Scharf C. A., 1997, *ApJ*, 418, L13
 Natarajan P., Hjorth J., van Kampen E., 1997, *MNRAS*, 286, 329
 Natarajan P., Kneib J.-P., Smail I., Ellis R. S., 1998, *ApJ*, 499, 600
 Navarro J. F., Frenk C. S., White S. D. M., 1997, *ApJ*, 490, 493 (NFW)
 Oukbir J., Blanchard A., 1997, *A&A*, 317, 1
 Pen U.-L., 1997, *New Astron.*, 2, 309
 Sadat R., Blanchard A., Oukbir J., 1998, *A&A*, 329, 21
 Sasaki S., 1996, *PASJ*, 48, L119
 Schindler S., 1996, *A&A*, 305, 756
 Smail I., Ellis R. S., Fitchett M. J., Edge A. C., 1995, *MNRAS*, 273, 277
 Smail I., Ellis R. S., Dressler A., Couch W. J., Oemler A., Sharples R. M., Butcher H., 1997, *ApJ*, 479, 70
 Squires G., Kaiser N., Babul A., Fahlman G., Woods D., Neumann D. M., Böhringer H., 1996a, *ApJ*, 461, 572
 Squires G., Kaiser N., Fahlman G., Babul A., Woods D., 1996b, *ApJ*, 449, 73
 Squires G., Neumann D. M., Kaiser N., Arnaud M., Babul A., Böhringer H., Fahlman G., Woods D., 1997, *ApJ*, 482, 648
 Taylor A. N., Dye S., Broadhurst T. J., Benítez N., van Kampen E., 1998, *ApJ*, in press (astro-ph/9801158)
 van Kampen E., 1998, *MNRAS*, submitted
 van Kampen E., Katgert P., 1997, *MNRAS*, 289, 327
 White S. D. M., Navarro J. F., Evrard A. E., Frenk C. S., 1993, *Nat*, 366, 429
 Yamashita K., 1997, in *ASCA/ROSAT Workshop on Clusters of Galaxies*. Japan Society for the Promotion of Science p. 71

This paper has been typeset from a $\text{T}_E\text{X}/\text{L}^A\text{T}_E\text{X}$ file prepared by the author.